Partial Space Time Dimensions
Non-Integered Metric Signatures

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Abstract

Summary of the article.

*Cheers!

Contents

1 Introduction

Few topics have been as fruitful to the development of modern Mathematics and Science than those that pertain to Space and Time.

Consider:

Definition 1.1 (Geometry)**.** The study of Shapes, Figures, and their underlying Spaces.

[1]

Definition 1.2 (Topology)**.** Studies [the] properties [of Spaces] that are invariant under deformation.

[2]

Seemingly innocuous and minor assumptions have sent the Scientific and Mathematical communities reeling when alternatives have been discovered: the denial of Euclidean Geometry and the development of Hyperbolic Geometry. Point Free Topologies, Relativity and modern treatments of Space Time, and so on.

2 Clarifications

I do not mean the representation of *a single dimension* (say via $\mathbb{N}$, $\mathbb{N}^+$, $\mathbb{Z}$, or $\mathbb{R}$), I mean *the quantity of* (commonly, *the number of*) Dimensions used to represent Space, Time, and/or SpaceTime. (Excluding for the moment the Dimensionality of Numbers, Self-Similarity, and the like. E.g. - the concept of Extension and Dimensionality as used in the course and tradition of ancient Western Philosophy and Maths.)

Such a notion (*the number of [Spatial] Dimensions*) is articulated and found present in a number of ways throughout the Mathematics literature:

- Via *Formal Definitions*: "A subspace M of a separable metric space X is zero-dimensional ..." [3]
- As **Cartesian Powers** (or **N-Fold Products**): $\mathbb{R}^4$ (e.g. - the 4)

[1] https://www.wolframalpha.com/examples/mathematics/geometry
[2] https://mathworld.wolfram.com/Topology.html
[3] Dimension Theory - RYSZARD ENGELKING - pp. 15

- Metric Signatures: $(3, 1)$
- The concept of: $n - dimensionality$
- The number of Elements within an *n-tuple*: (a, b, c)

2.1 R-Dimensionality

I adopt the convvention that *dimensionality* is given by the following:

Definition 2.1 (Dimensionality)**.** The *number or quantity of Dimensions* within a **Space**. And, **not** the way *a single* Dimension is represented.

(The latter we shall henceforth refer to as *"representation of a dimension"* - how a single dimension is represented - to avoid confusion.)

From that, I adopt the hopefully less confusing distinction between *n-dimensionality* and *r-dimensionality* to make the notion at hand more precise.

Definition 2.2 (R-Dimensionality)**.** Where the number of **Dimesions** of a **Space** is R and $R \in \mathbb{R}$. And, where the **Points** constituting such a **Space** are given by **R-Fold Products** and $R \in \mathbb{R}$.

Some examples of *r-dimensionality*:

- A Euclidean space with 1.11 dimensions where each dimension is Natural Number-intervaled.
- A pseudo-Euclidean space with 0.99999999... dimensions where each dimension is Real-intervaled.
- A Riemannian manifold, Minkowski space with 3.14 dimensions where each dimension is Real-intervaled.
- A Euclidean space with 2.5 dimensions where each dimension is Real-intervaled.
- A Lorentzian Manifold with dimension $r >= 2$, r is not an Ordinal with metric signature $(r, 1)$.
- A Minkowski Spacetime with signature $(2.999...., 1)$ such that the underlying smooth Euclidean space is 2.999... dimensioned.

3 Current and Historical Conceptions of Space and Time

3.1 Relationalism vs. Absolutism

Netwon and Leibniz

Are we still enraptured by Kantian conceptions of Spatiality?

3.2 Euclid and Riemann

Yeah those guys are pretty famous and helpful in history for humanity, science, and the like.

3.3 Isometric Perspective and 1.5D Graphics

Graphics Rendering

A perspective, projection, or transformation but still using the **Integered Conception** underneath

4 Existing Dimensional Notions in Maths

Topological and Small Inductive Topological Spaces

[4]

4.1 Hausdorff Dimensions

Measures self-similarity (fractals) and uses the **Integered Conception**

4.2 Metric Signatures

5 Contra Descartes

Here's a David vs. Goliath potshot...

Suprising how influential Renee Descartes still is. Every formulation of Space, Topological Spaces, and so on still uses **Cartesian Multiplication**

[4]see: footnote

$\{A, B, C\} \times \{D, E, F\} =$
$$\{(A, D), (A, E), (A, F), (B, D), (B, E), (B, F), (C, D), (C, E), (C, F)\}$$

So, in line with my general attack on "Early Modern" Philosophy, I will attempt to supplement this long reigning dogma with a viable alternative.

5.1 Partial and Fuzzy Sets

Partial Elemental Inclusion operator.

Definition 5.1 (Partial Elemental Inclusion)**.** A's being an element of B is mapped to the interval $\big[0, 1\big]$ $(A \in B) \rightarrow \big[0, 1\big]$

We might think of Fuzzy Sets and Partial Elemental Inclusion as Models or representations of Partial Dimensions.

- Say, $(A \in B) = .5$ represents .5 Dimensions.
- Or, $(A \in B) = .11111...$ represents .11111... Dimensions.

Is this sufficient to capture partial Dimensions? Or just a single conception?
[5]

5.2 R-Tuples

Better distinction between n-tuples and r-tuples (where r is indexed the Reals - e.g. $X = (a, b, c, ..., r)$ where X has say $r - tuples$ where $r = 2.111$.

6 Dimensions With Blurry Boundaries

The **Integered Conception** is parasitic on the idea that Dimensions are "sharply bounded" ("carved at the joints" is common-enough parlance).

Does the First Dimension bleed into the Second (in some sense)? Can one in fact "climb" Dimensions on such a conception? We might ask, it what meaningful way are they distinct?

[5] Add some notes from: https://www.thoughtscript.io/blog/000000000135 and elsewhere

7 Some Other Notions

7.1 Holographic Shadowing

A spatial object that's defined to exist in some Dimensions but not others (it skips or eludes certain Dimensions).

8 Conclusion

References

[1] First, J. and Second, J. (2010), "An interesting paper", *A Famous Journal*, 1, pp. 1–11.

[2] First, J. and Second, J. (2011), *A great book*, Address: Publisher.